

## Four-Neutrino Mixing

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### Abstract

It is shown that at least four massive neutrinos are needed in order to accommodate the evidences in favor of neutrino oscillations found in solar and atmospheric neutrino experiments and in the LSND experiment. Among all four-neutrino schemes, only two are compatible with the results of all neutrino oscillation experiments. These two schemes have a mass spectrum composed of two pairs of neutrinos with close masses separated by the “LSND gap” of the order of 1 eV.

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## I. INTRODUCTION

Neutrino oscillations have been proposed by B. Pontecorvo [1] more than forty years ago following an analogy with  $K^0 \rightleftharpoons \bar{K}^0$  oscillations. Today this beautiful quantum mechanical phenomenon [2,3] is subject to intensive experimental and theoretical research. Besides the intrinsic interest related to the investigation of the fundamental properties of neutrinos, it is considered to be one of the best ways to explore the physics beyond the Standard Model.

The best evidence in favor of the existence of neutrino oscillations has been recently provided by the measurement in the Super-Kamiokande experiment [4] of an up-down asymmetry of high-energy  $\mu$ -like events generated by atmospheric neutrinos:

$$\mathcal{A}_\mu \equiv (D_\mu - U_\mu)/(D_\mu + U_\mu) = 0.311 \pm 0.043 \pm 0.01. \quad (1.1)$$

Here  $D_\mu$  and  $U_\mu$  are, respectively, the number of downward-going and upward-going events, corresponding to the zenith angle intervals  $0.2 < \cos \theta < 1$  and  $-1 < \cos \theta < -0.2$ . Since the fluxes of high-energy downward-going and upward-going atmospheric neutrinos are predicted to be equal with high accuracy on the basis of geometrical arguments [5], the Super-Kamiokande evidence in favor of neutrino oscillations is model-independent and provides a confirmation of the indications in favor of oscillations of atmospheric neutrinos found in the Super-Kamiokande experiment itself [4,6] and in other experiments through the measurement of the ratio of  $\mu$ -like and  $e$ -like events (Kamiokande, IMB, Soudan 2) [7] and through the measurement of upward-going muons produced by neutrino interactions in the rock below the detector (MACRO) [8]. Large  $\nu_\mu \rightleftharpoons \nu_e$  oscillations of atmospheric neutrinos are excluded by the absence of a up-down asymmetry of high-energy  $e$ -like events generated by atmospheric neutrinos and detected in the Super-Kamiokande experiment ( $\mathcal{A}_e = 0.036 \pm 0.067 \pm 0.02$ ) [4] and by the negative result of the CHOOZ long-baseline  $\bar{\nu}_e$  disappearance experiment [9]. Therefore, the atmospheric neutrino anomaly consists in the disappearance of muon neutrinos and can be explained by  $\nu_\mu \rightarrow \nu_\tau$  and/or  $\nu_\mu \rightarrow \nu_s$  oscillations (here  $\nu_s$  is a sterile neutrino that does not take part in weak interactions).

Other indications in favor of neutrino oscillations have been obtained in solar neutrino experiments (Homestake, Kamiokande, GALLEX, SAGE, Super-Kamiokande) [10] and in the LSND experiment [11].

The flux of electron neutrinos measured in all five solar neutrino experiments is substantially smaller than the one predicted by the Standard Solar Model [12] and a comparison of the data of different experiments indicate an energy dependence of the solar  $\nu_e$  suppression, which represents a rather convincing evidence in favor of neutrino oscillations [3]. The disappearance of solar electron neutrinos can be explained by  $\nu_e \rightarrow \nu_\mu$  and/or  $\nu_e \rightarrow \nu_\tau$  and/or  $\nu_e \rightarrow \nu_s$  oscillations [13].

The accelerator LSND experiment is the only one that claims the observation of neutrino oscillations in specific appearance channels:  $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$  and  $\nu_\mu \rightarrow \nu_e$ . Since the appearance of neutrinos with a different flavor represents the true essence of neutrino oscillations, the LSND evidence is extremely interesting and its confirmation (or disproof) by other experiments should receive high priority in future research. Four such experiments have been proposed and are under study: BooNE at Fermilab, I-216 at CERN, ORLaND at Oak Ridge and NESS at the European Spallation Source [14]. Among these proposals only BooNE is approved and will start in 2001.

Neutrino oscillations occur if neutrinos are massive and mixed particles [2,3], *i.e.* if the left-handed components  $\nu_{\alpha L}$  of the flavor neutrino fields are superpositions of the left-handed components  $\nu_{kL}$  ( $k = 1, \dots, N$ ) of neutrino fields with definite mass  $m_k$ :

$$\nu_{\alpha L} = \sum_{k=1}^N U_{\alpha k} \nu_{kL}, \quad (1.2)$$

where  $U$  is a  $N \times N$  unitary mixing matrix. From the measurement of the invisible decay width of the  $Z$ -boson it is known that the number of light active neutrino flavors is three [15], corresponding to  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$  (active neutrinos are those taking part to standard weak interactions). This implies that the number  $N$  of massive neutrinos is bigger or equal to three. If  $N > 3$ , in the flavor basis there are  $N_s = N - 3$  sterile neutrinos,  $\nu_{s_1}, \dots, \nu_{s_{N_s}}$ , that do not take part to standard weak interactions. In this case the flavor index  $\alpha$  in Eq. (1.2) takes the values  $e, \mu, \tau, s_1, \dots, s_{N_s}$ .

## II. THE NECESSITY OF AT LEAST THREE INDEPENDENT $\Delta M^2$ 'S

The three evidences in favor of neutrino oscillations found in solar and atmospheric neutrino experiments and in the accelerator LSND experiment imply the existence of at least three independent neutrino mass-squared differences. This can be seen by considering the general expression for the probability of  $\nu_\alpha \rightarrow \nu_\beta$  transitions in vacuum, that can be written as [2,3]

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \left| \sum_{k=1}^N U_{\alpha k}^* U_{\beta k} \exp \left( -i \frac{\Delta m_{kj}^2 L}{2 E} \right) \right|^2, \quad (2.1)$$

where  $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$ ,  $j$  is any of the mass-eigenstate indices,  $L$  is the distance between the neutrino source and detector and  $E$  is the neutrino energy. The range of  $L/E$  characteristic of each type of experiment is different:  $L/E \sim 10^{11} - 10^{12} \text{ eV}^{-2}$  for solar neutrino experiments,  $L/E \sim 10^2 - 10^3 \text{ eV}^{-2}$  for atmospheric neutrino experiments and  $L/E \sim 1 \text{ eV}^{-2}$  for the LSND experiment. From Eq. (2.1) it is clear that neutrino oscillations are observable in an experiment only if there is at least one mass-squared difference  $\Delta m_{kj}^2$  such that

$$\frac{\Delta m_{kj}^2 L}{2 E} \gtrsim 0.1 \quad (2.2)$$

(the precise lower bound depends on the sensitivity of the experiment) in a significant part of the energy and source-detector distance intervals of the experiment (if the condition (2.2) is not satisfied,  $P_{\nu_\alpha \rightarrow \nu_\beta} \simeq |\sum_k U_{\alpha k}^* U_{\beta k}|^2 = \delta_{\alpha\beta}$ ). Since the range of  $L/E$  probed by the LSND experiment is the smaller one, a large mass-squared difference is needed for LSND oscillations:

$$\Delta m_{\text{LSND}}^2 \gtrsim 10^{-1} \text{ eV}^2. \quad (2.3)$$

Specifically, the maximum likelihood analysis of the LSND data in terms of two-neutrino oscillations gives [11]

$$0.20 \text{ eV}^2 \lesssim \Delta m_{\text{LSND}}^2 \lesssim 2.0 \text{ eV}^2. \quad (2.4)$$

Furthermore, from Eq. (2.1) it is clear that a dependence of the oscillation probability from the neutrino energy  $E$  and the source-detector distance  $L$  is observable only if there is at least one mass-squared difference  $\Delta m_{kj}^2$  such that

$$\frac{\Delta m_{kj}^2 L}{2 E} \sim 1. \quad (2.5)$$

Indeed, all the phases  $\Delta m_{kj}^2 L/2E \gg 1$  are washed out by the average over the energy and source-detector ranges characteristic of the experiment. Since a variation of the oscillation probability as a function of neutrino energy has been observed both in solar and atmospheric neutrino experiments and the ranges of  $L/E$  characteristic of these two types of experiments are different from each other and different from the LSND range, two more mass-squared differences with different scales are needed:

$$\Delta m_{\text{sun}}^2 \sim 10^{-12} - 10^{-11} \text{ eV}^2 \quad (\text{VO}), \quad (2.6)$$

$$\Delta m_{\text{atm}}^2 \sim 10^{-3} - 10^{-2} \text{ eV}^2. \quad (2.7)$$

The condition (2.6) for the solar mass-squared difference  $\Delta m_{\text{sun}}^2$  has been obtained under the assumption of vacuum oscillations (VO). If the disappearance of solar  $\nu_e$ 's is due to the MSW effect [16], the condition

$$\Delta m_{\text{sun}}^2 \lesssim 10^{-4} \text{ eV}^2 \quad (\text{MSW}) \quad (2.8)$$

must be fulfilled in order to have a resonance in the interior of the sun. Hence, in the MSW case  $\Delta m_{\text{sun}}^2$  must be at least one order of magnitude smaller than  $\Delta m_{\text{atm}}^2$ .

It is possible to ask if three different scales of neutrino mass-squared differences are needed even if the results of the Homestake solar neutrino experiment is neglected, allowing an energy-independent suppression of the solar  $\nu_e$  flux. The answer is that still the data cannot be fitted with only two neutrino mass-squared differences because an energy-independent suppression of the solar  $\nu_e$  flux requires large  $\nu_e \rightarrow \nu_\mu$  or  $\nu_e \rightarrow \nu_\tau$  transitions generated by  $\Delta m_{\text{atm}}^2$  or  $\Delta m_{\text{LSND}}^2$ . These transitions are forbidden by the results of the Bugey [17] and CHOOZ [9] reactor  $\bar{\nu}_e$  disappearance experiments and by the non-observation of an up-down asymmetry of  $e$ -like events in the Super-Kamiokande atmospheric neutrino experiment [4].

### III. FOUR-NEUTRINO SCHEMES

The existence of three different scales of  $\Delta m^2$  imply that at least four light massive neutrinos must exist in nature. Here we consider the schemes with four light and mixed neutrinos [18–33], which constitute the minimal possibility that allows to explain all the existing data with neutrino oscillations. In this case, in the flavor basis the three active neutrinos  $\nu_e$ ,  $\nu_\mu$ ,  $\nu_\tau$  are accompanied by a sterile neutrino  $\nu_s$  that does not take part in standard weak interactions.

The six types of four-neutrino mass spectra with three different scales of  $\Delta m^2$  that can accommodate the hierarchy  $\Delta m_{\text{sun}}^2 \ll \Delta m_{\text{atm}}^2 \ll \Delta m_{\text{LSND}}^2$  are shown qualitatively in Fig. 1.

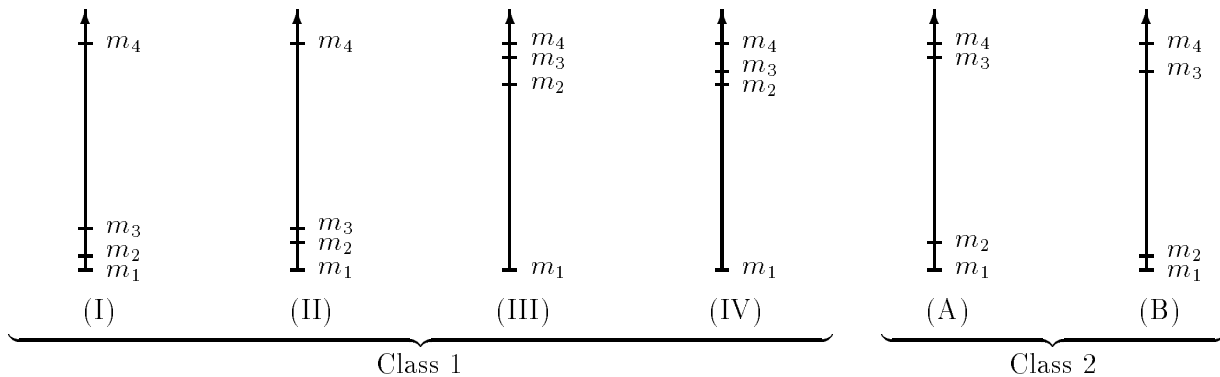


Figure 1

In all these mass spectra there are two groups of close masses separated by the “LSND gap” of the order of 1 eV. In each scheme the smallest mass-squared difference corresponds to  $\Delta m_{\text{sun}}^2$  ( $\Delta m_{21}^2$  in schemes I and B,  $\Delta m_{32}^2$  in schemes II and IV,  $\Delta m_{43}^2$  in schemes III and A), the intermediate one to  $\Delta m_{\text{atm}}^2$  ( $\Delta m_{31}^2$  in schemes I and II,  $\Delta m_{42}^2$  in schemes III and IV,  $\Delta m_{21}^2$  in scheme A,  $\Delta m_{43}^2$  in scheme B) and the largest mass squared difference  $\Delta m_{41}^2 = \Delta m_{\text{LSND}}^2$  is relevant for the oscillations observed in the LSND experiment. The six schemes are divided into four schemes of class 1 (I–IV) in which there is a group of three masses separated from an isolated mass by the LSND gap, and two schemes of class 2 (A, B) in which there are two couples of close masses separated by the LSND gap.

#### IV. THE DISFAVORED SCHEMES OF CLASS 1

In the following we will show that the schemes of class 1 are disfavored by the data if also the negative results of short-baseline accelerator and reactor disappearance neutrino oscillation experiments are taken into account [21,27,28]. Let us remark that in principle one could check which schemes are allowed by doing a combined fit of all data in the framework of the most general four-neutrino mixing scheme, with three mass-squared differences, six mixing angles and three CP-violating phases as free parameters. However, at the moment it is not possible to perform such a fit because of the enormous complications due to the presence of too many parameters and to the difficulties involved in a combined fit of the data of different experiments, which are usually analyzed by the experimental collaborations using different methods. Hence, we think that it is quite remarkable that one can exclude the schemes of class 1 with the following relatively simple procedure.

Let us define the quantities  $d_\alpha$ , with  $\alpha = e, \mu, \tau, s$ , in the schemes of class 1 as

$$d_\alpha^{(\text{I})} \equiv |U_{\alpha 4}|^2, \quad d_\alpha^{(\text{II})} \equiv |U_{\alpha 4}|^2, \quad d_\alpha^{(\text{III})} \equiv |U_{\alpha 1}|^2, \quad d_\alpha^{(\text{IV})} \equiv |U_{\alpha 1}|^2. \quad (4.1)$$

Physically  $d_\alpha$  quantifies the mixing of the flavor neutrino  $\nu_\alpha$  with the isolated neutrino, whose mass is separated from the other three by the LSND gap.

The probability of  $\nu_\alpha \rightarrow \nu_\beta$  ( $\beta \neq \alpha$ ) and  $\nu_\alpha \rightarrow \nu_\alpha$  transitions (and the corresponding probabilities for antineutrinos) in short-baseline experiments are given by [21,3]

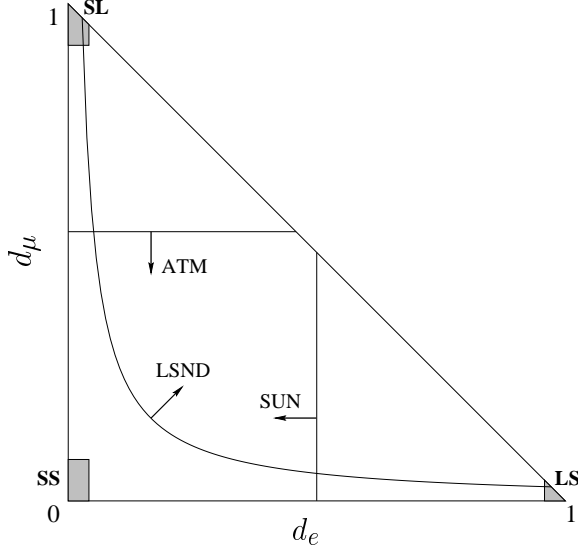


Figure 2

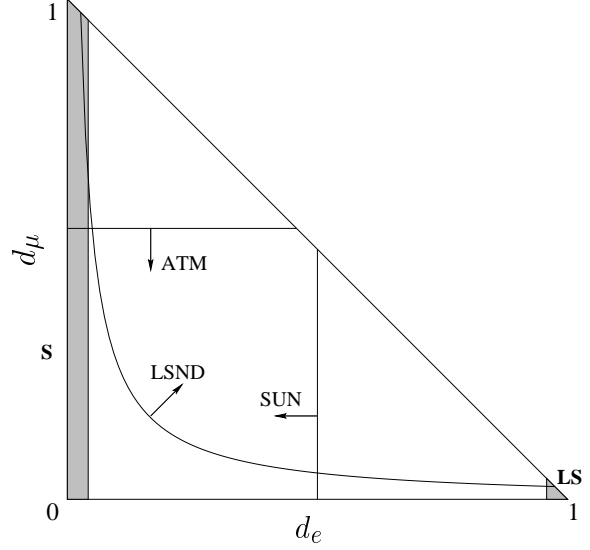


Figure 3

$$P_{\nu_\alpha \rightarrow \nu_\beta} = A_{\alpha;\beta} \sin^2 \frac{\Delta m_{41}^2 L}{4E}, \quad P_{\nu_\alpha \rightarrow \nu_\alpha} = 1 - B_{\alpha;\alpha} \sin^2 \frac{\Delta m_{41}^2 L}{4E}, \quad (4.2)$$

with the oscillation amplitudes

$$A_{\alpha;\beta} = 4 d_\alpha d_\beta, \quad B_{\alpha;\alpha} = 4 d_\alpha (1 - d_\alpha). \quad (4.3)$$

The probabilities (4.2) have the same form as the corresponding probabilities in the case of two-neutrino mixing,  $P_{\nu_\alpha \rightarrow \nu_\beta} = \sin^2(2\vartheta) \sin^2(\Delta m^2 L/4E)$  and  $P_{\nu_\alpha \rightarrow \nu_\alpha} = 1 - \sin^2(2\vartheta) \sin^2(\Delta m^2 L/4E)$ , which have been used by all experimental collaborations for the analysis of the data in order to get information on the parameters  $\sin^2(2\vartheta)$  and  $\Delta m^2$  ( $\vartheta$  and  $\Delta m^2$  are, respectively, the mixing angle and the mass-squared difference in the case of two-neutrino mixing). Therefore, we can use the results of their analyses in order to get information on the corresponding parameters  $A_{\alpha;\beta}$ ,  $B_{\alpha;\alpha}$  and  $\Delta m_{41}^2$ .

The exclusion plots obtained in short-baseline  $\bar{\nu}_e$  and  $\nu_\mu$  disappearance experiments imply that [21]

$$d_\alpha \leq a_\alpha^0 \quad \text{or} \quad d_\alpha \geq 1 - a_\alpha^0 \quad (\alpha = e, \mu), \quad (4.4)$$

with

$$a_\alpha^0 = \frac{1}{2} \left( 1 - \sqrt{1 - B_{\alpha;\alpha}^0} \right) \quad (\alpha = e, \mu), \quad (4.5)$$

where  $B_{e;e}^0$  and  $B_{\mu;\mu}^0$  are the upper bounds, that depend on  $\Delta m_{41}^2$ , of the oscillation amplitudes  $B_{e;e}$  and  $B_{\mu;\mu}$  given by the exclusion plots of  $\bar{\nu}_e$  and  $\nu_\mu$  disappearance experiments. From the exclusion curves of the Bugey reactor  $\bar{\nu}_e$  disappearance experiment [17] and of the CDHS and CCFR accelerator  $\nu_\mu$  disappearance experiments [34] it follows that  $a_e^0 \lesssim 3 \times 10^{-2}$  for  $\Delta m_{41}^2 = \Delta m_{\text{LSND}}^2$  in the LSND range (2.4) and  $a_\mu^0 \lesssim 0.2$  for  $\Delta m_{41}^2 \gtrsim 0.4 \text{ eV}^2$  [3].

Therefore, the negative results of short-baseline  $\bar{\nu}_e$  and  $\nu_\mu$  disappearance experiments imply that  $d_e$  and  $d_\mu$  are either small or large (close to one). Taking into account the

unitarity limit  $d_e + d_\mu \leq 1$ , for each value of  $\Delta m_{41}^2$  above about  $0.3 \text{ eV}^2$  there are three regions in the  $d_e$ - $d_\mu$  plane that are allowed by the results of disappearance experiments: region SS with small  $d_e$  and  $d_\mu$ , region LS with large  $d_e$  and small  $d_\mu$  and region SL with small  $d_e$  and large  $d_\mu$ . These three regions are illustrated qualitatively by the three shadowed areas in Fig. 2. For  $\Delta m_{41}^2 \lesssim 0.3 \text{ eV}^2$  there is no constraint on the value of  $d_\mu$  from the results of short-baseline  $\nu_\mu$  disappearance experiments and there are two regions in the  $d_e$ - $d_\mu$  plane that are allowed by the results of  $\bar{\nu}_e$  disappearance experiments: region S with small  $d_e$  and region LS with large  $d_e$  and small  $d_\mu$  (the smallness of  $d_\mu$  follows from the unitarity bound  $d_e + d_\mu \leq 1$ ). These two regions are illustrated qualitatively by the two shadowed areas in Fig. 3.

Let us consider now the results of solar neutrino experiments, which imply a disappearance of electron neutrinos. The survival probability of solar  $\nu_e$ 's averaged over the fast unobservable oscillations due to  $\Delta m_{41}^2$  and  $\Delta m_{31}^2$  is bounded by [21,3]

$$P_{\nu_e \rightarrow \nu_e}^{\text{sun}} \geq d_e^2. \quad (4.6)$$

Therefore, only the possibility

$$d_e \leq a_e^0 \quad (4.7)$$

is acceptable in order to explain the observed deficit of solar  $\nu_e$ 's with neutrino oscillations. Indeed, the solar neutrino data imply an upper bound for  $d_e$ , that is shown qualitatively by the vertical lines in Figs. 2 and 3. It is clear that the regions LS in Figs. 2 and 3 are disfavored by the results of solar neutrino experiments.

In a similar way, since the survival probability of atmospheric  $\nu_\mu$ 's and  $\bar{\nu}_\mu$ 's is bounded by [21,3]

$$P_{\nu_\mu \rightarrow \nu_\mu}^{\text{atm}} \geq d_\mu^2, \quad (4.8)$$

large values of  $d_\mu$  are incompatible with the asymmetry (1.1) observed in the Super-Kamiokande experiment. The upper bound for  $d_\mu$  that follows from atmospheric neutrino data is shown qualitatively by the horizontal lines in Figs. 2 and 3. It is clear that the region SL in Fig. 2, that is allowed by the results of  $\nu_\mu$  short-baseline disappearance experiments for  $\Delta m_{41}^2 \gtrsim 0.3 \text{ eV}^2$ , and the large- $d_\mu$  part of the region S in Fig. 3 are disfavored by the results of atmospheric neutrino experiments. A precise calculation [28] shows that the Super-Kamiokande asymmetry (1.1) and the exclusion curve of the Bugey  $\bar{\nu}_e$  disappearance experiment imply the upper bound

$$d_\mu \lesssim 0.55 \equiv a_\mu^{\text{SK}}. \quad (4.9)$$

This upper bound is depicted by the horizontal line in Fig. 4 (the vertically hatched area above the line is excluded).

In Fig. 4 we have also shown the bound  $d_\mu \leq a_\mu^0$  or  $d_\mu \geq 1 - a_\mu^0$  obtained from the exclusion plot of the short-baseline CDHS  $\nu_\mu$  disappearance experiment, which exclude the shadowed region. It is clear that the results of short-baseline disappearance experiments and the Super-Kamiokande asymmetry (1.1) imply that  $d_\mu \lesssim 0.55$  for  $\Delta m_{41}^2 \lesssim 0.3 \text{ eV}^2$  and that  $d_\mu$  is very small for  $\Delta m_{41}^2 \gtrsim 0.3 \text{ eV}^2$ . However, this range of  $d_\mu$  is disfavored by the results

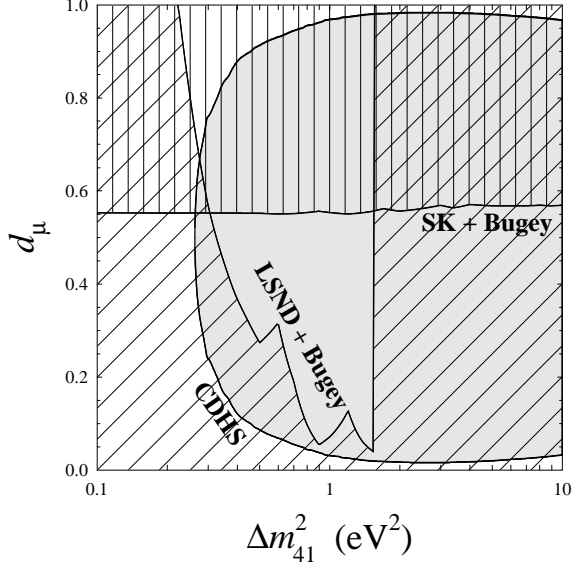


Figure 4

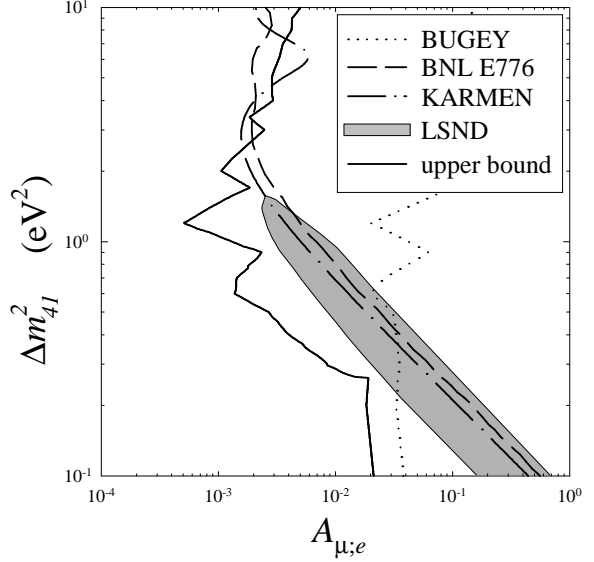


Figure 5

of the LSND experiment, that imply a lower bound  $A_{\mu;e}^{\min}$  for the amplitude  $A_{\mu;e} = 4d_e d_\mu$  of  $\nu_\mu \rightarrow \nu_e$  oscillations. Indeed, we have

$$d_e d_\mu \geq A_{\mu;e}^{\min} / 4. \quad (4.10)$$

This bound, shown qualitatively by the LSND exclusion curves in Figs. 2 and 3, excludes region SS in Fig. 2 and the small- $d_\mu$  part of region S in Fig. 3. From Figs. 2 and 3 one can see in a qualitative way that in the schemes of class 1 the results of the solar, atmospheric and LSND experiments are incompatible with the negative results of short-baseline experiments.

A quantitative illustration of this incompatibility is given in Fig. 4. The curve in Fig. 4 labelled LSND + Bugey (the diagonally hatched area is excluded) represents the constraint

$$d_\mu \geq A_{\mu;e}^{\min} / 4a_e^0, \quad (4.11)$$

derived from the inequality (4.10) using the bound (4.7). One can see that the results of the LSND experiment exclude the range of  $d_\mu$  allowed by the results of short-baseline disappearance experiments and by the Super-Kamiokande asymmetry (1.1). Hence, in the framework of the schemes of class 1 there is no range of  $d_\mu$  that is compatible with all the experimental data.

The incompatibility of the experimental results with the schemes of class 1 is shown also in Fig. 5, where we have plotted in the  $A_{\mu;e}-\Delta m_{41}^2$  plane the upper bound  $A_{\mu;e} \leq 4a_e^0 a_\mu^0$  for  $\Delta m_{41}^2 > 0.26 \text{ eV}^2$  and  $A_{\mu;e} \leq 4a_e^0 a_\mu^{\text{SK}}$  for  $\Delta m_{41}^2 < 0.26 \text{ eV}^2$  (solid line, the region on the right is excluded). One can see that this constraint is incompatible with the LSND-allowed region (shaded area).

Summarizing, we have reached the conclusion that the four schemes of class 1 shown in Fig. 1 are disfavored by the data.



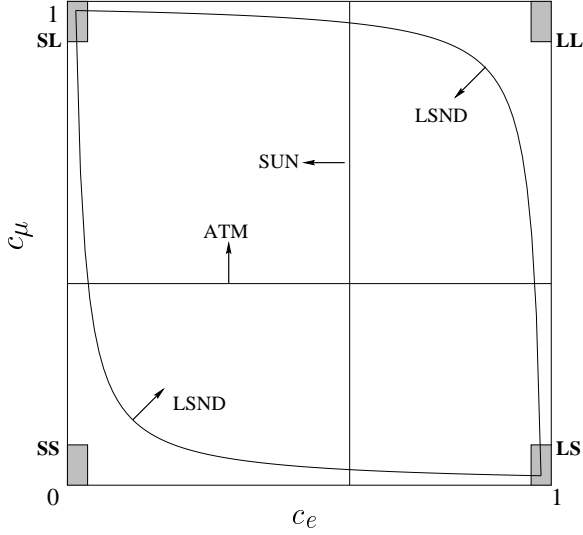


Figure 6

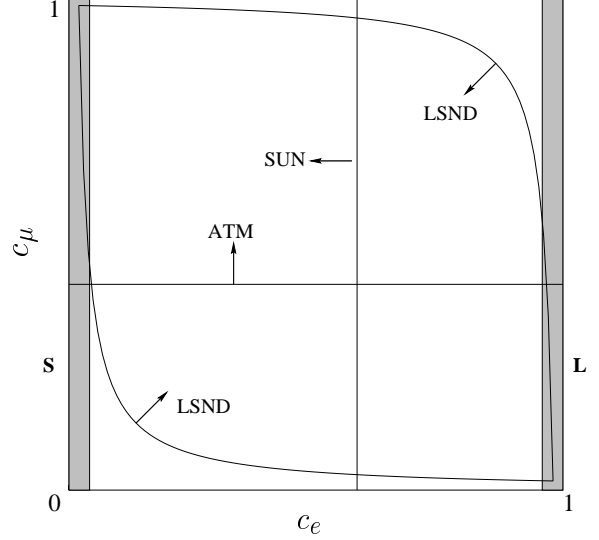


Figure 7

## V. THE FAVORED SCHEMES OF CLASS 2

The four-neutrino schemes of class 2 are compatible with the results of all neutrino oscillation experiments if the mixing of  $\nu_e$  with the two mass eigenstates responsible for the oscillations of solar neutrinos ( $\nu_3$  and  $\nu_4$  in scheme A and  $\nu_1$  and  $\nu_2$  in scheme B) is large and the mixing of  $\nu_\mu$  with the two mass eigenstates responsible for the oscillations of atmospheric neutrinos ( $\nu_1$  and  $\nu_2$  in scheme A and  $\nu_3$  and  $\nu_4$  in scheme B) is large [20,21,27,28]. This is illustrated qualitatively in Figs. 6 and 7, as we are going to explain.

Let us define the quantities  $c_\alpha$ , with  $\alpha = e, \mu, \tau, s$ , in the schemes A and B as

$$c_\alpha^{(A)} \equiv \sum_{k=1,2} |U_{\alpha k}|^2, \quad c_\alpha^{(B)} \equiv \sum_{k=3,4} |U_{\alpha k}|^2. \quad (5.1)$$

Physically  $c_\alpha$  quantify the mixing of the flavor neutrino  $\nu_\alpha$  with the two massive neutrinos whose  $\Delta m^2$  is relevant for the oscillations of atmospheric neutrinos ( $\nu_1, \nu_2$  in scheme A and  $\nu_3, \nu_4$  in scheme B). The exclusion plots obtained in short-baseline  $\bar{\nu}_e$  and  $\nu_\mu$  disappearance experiments imply that [21]

$$c_\alpha \leq a_\alpha^0 \quad \text{or} \quad c_\alpha \geq 1 - a_\alpha^0 \quad (\alpha = e, \mu), \quad (5.2)$$

with  $a_\alpha^0$  given in Eq. (4.5).

The shadowed areas in Figs. 6 and 7 illustrate qualitatively the regions in the  $c_e$ - $c_\mu$  plane allowed by the negative results of short-baseline  $\bar{\nu}_e$  and  $\nu_\mu$  disappearance experiments. Figure 6 is valid for  $\Delta m_{41}^2 \gtrsim 0.3 \text{ eV}^2$  and shows that there are four regions allowed by the results of short-baseline disappearance experiments: region SS with small  $c_e$  and  $c_\mu$ , region LS with large  $c_e$  and small  $c_\mu$ , region SL with small  $c_e$  and large  $c_\mu$  and region LL with large  $c_e$  and  $c_\mu$ . The quantities  $c_e$  and  $c_\mu$  can be both large, because the unitarity of the mixing matrix imply that  $c_\alpha + c_\beta \leq 2$  and  $0 \leq c_\alpha \leq 1$  for  $\alpha, \beta = e, \mu, \tau, s$ . Figure 7 is valid for  $\Delta m_{41}^2 \lesssim 0.3 \text{ eV}^2$ , where there is no constraint on the value of  $c_\mu$  from the results

of short-baseline  $\nu_\mu$  disappearance experiments. It shows that there are two regions allowed by the results of short-baseline  $\bar{\nu}_e$  disappearance experiments: region S with small  $c_e$  and region L with large  $c_e$ .

Let us take now into account the results of solar neutrino experiments. Large values of  $c_e$  are incompatible with solar neutrino oscillations because in this case  $\nu_e$  has large mixing with the two massive neutrinos responsible for atmospheric neutrino oscillations and, through the unitarity of the mixing matrix, small mixing with the two massive neutrinos responsible for solar neutrino oscillations. Indeed, in the schemes of class 2 the survival probability  $P_{\nu_e \rightarrow \nu_e}^{\text{sun}}$  of solar  $\nu_e$ 's is bounded by [21,3]

$$P_{\nu_e \rightarrow \nu_e}^{\text{sun}} \geq c_e^2/2, \quad (5.3)$$

and its possible variation  $\Delta P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E)$  with neutrino energy  $E$  is limited by [21,3]

$$\Delta P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E) \leq (1 - c_e)^2. \quad (5.4)$$

If  $c_e$  is large as in the LS or LL regions of Fig. 6 or in the L region of Fig. 7, we have

$$P_{\nu_e \rightarrow \nu_e}^{\text{sun}} \geq \frac{(1 - a_e^0)^2}{2} \simeq \frac{1}{2}, \quad \Delta P_{\nu_e \rightarrow \nu_e}^{\text{sun}}(E) \leq (a_e^0)^2 \lesssim 10^{-3}, \quad (5.5)$$

for  $\Delta m_{41}^2 = \Delta m_{\text{LSND}}^2$  in the LSND range (2.4). Therefore  $P_{\nu_e \rightarrow \nu_e}^{\text{sun}}$  is bigger than 1/2 and practically does not depend on neutrino energy. Since this is incompatible with the results of solar neutrino experiments interpreted in terms of neutrino oscillations [3,13], we conclude that the regions LS and LL in Fig. 6 and the region L in Fig. 7 are disfavored by solar neutrino data, as illustrated qualitatively by the vertical exclusion lines in Figs. 6 and 7.

Let us consider now the results of atmospheric neutrino experiments. Small values of  $c_\mu$  are incompatible with atmospheric neutrino oscillations because in this case  $\nu_\mu$  has small mixing with the two massive neutrinos responsible for atmospheric neutrino oscillations. Indeed, the survival probability of atmospheric  $\nu_\mu$ 's is bounded by [21,3]

$$P_{\nu_\mu \rightarrow \nu_\mu}^{\text{atm}} \geq (1 - c_\mu)^2, \quad (5.6)$$

and it can be shown [28] that the Super-Kamiokande asymmetry (1.1) and the exclusion curve of the Bugey  $\bar{\nu}_e$  disappearance experiment imply the upper bound

$$c_\mu \gtrsim 0.45 \equiv b_\mu^{\text{SK}}. \quad (5.7)$$

This limit is depicted qualitatively by the horizontal exclusion lines in Figs. 6 and 7. Therefore, we conclude that the regions SS and LS in Fig. 6 and the small- $c_\mu$  parts of the regions S and L in Fig. 7 are disfavored by atmospheric neutrino data.

Finally, let us consider the results of the LSND experiment. In the schemes of class 2 the amplitude of short-baseline  $\nu_\mu \rightarrow \nu_e$  oscillations is given by

$$A_{\mu;e} = \left| \sum_{k=1,2} U_{ek} U_{\mu k}^* \right|^2 = \left| \sum_{k=3,4} U_{ek} U_{\mu k}^* \right|^2. \quad (5.8)$$

The second equality in Eq. (5.8) is due to the unitarity of the mixing matrix. Using the Cauchy–Schwarz inequality we obtain

$$c_e c_\mu \geq A_{\mu;e}^{\min}/4 \quad \text{and} \quad (1 - c_e)(1 - c_\mu) \geq A_{\mu;e}^{\min}/4, \quad (5.9)$$

where  $A_{\mu;e}^{\min}$  is the minimum value of the oscillation amplitude  $A_{\mu;e}$  observed in the LSND experiment. The bounds (5.9) are illustrated qualitatively in Figs. 6 and 7. One can see that the results of the LSND experiment confirm the exclusion of the regions SS and LL in Fig. 6 and the exclusion of the small- $c_\mu$  part of region S and of the large- $c_\mu$  part of region L in Fig. 7.

Summarizing, if  $\Delta m_{41}^2 \gtrsim 0.3 \text{ eV}^2$  only the region SL in Fig. 6, with

$$c_e \leq a_e^0 \quad \text{and} \quad c_\mu \geq 1 - a_\mu^0, \quad (5.10)$$

is compatible with the results of all neutrino oscillation experiments. If  $\Delta m_{41}^2 \lesssim 0.3 \text{ eV}^2$  only the large- $c_\mu$  part of region S in Fig. 7, with

$$c_e \leq a_e^0 \quad \text{and} \quad c_\mu \geq b_\mu^{\text{SK}}, \quad (5.11)$$

is compatible with the results of all neutrino oscillation experiments. Therefore, in any case  $c_e$  is small and  $c_\mu$  is large. However, it is important to notice that, as shown clearly in Figs. 6 and 7, the inequalities (5.9) following from the LSND observation of short-baseline  $\nu_\mu \rightarrow \nu_e$  oscillations imply that  $c_e$ , albeit small, has a lower bound and  $c_\mu$ , albeit large, has an upper bound:

$$c_e \gtrsim A_{\mu;e}^{\min}/4 \quad \text{and} \quad c_\mu \lesssim 1 - A_{\mu;e}^{\min}/4. \quad (5.12)$$

## VI. CONCLUSIONS

We have seen that only the two four-neutrino schemes A and B of class 2 in Fig. 1 are compatible with the results of all neutrino oscillation experiments. Furthermore, we have shown that the quantities  $c_e$  and  $c_\mu$  in these two schemes must be, respectively, small and large. Physically  $c_\alpha$ , defined in Eq. (5.1), quantify the mixing of the flavor neutrino  $\nu_\alpha$  with the two massive neutrinos whose  $\Delta m^2$  is relevant for the oscillations of atmospheric neutrinos ( $\nu_1, \nu_2$  in scheme A and  $\nu_3, \nu_4$  in scheme B).

The smallness of  $c_e$  implies that electron neutrinos do not oscillate in atmospheric and long-baseline neutrino oscillation experiments. Indeed, one can obtain rather stringent upper bounds for the probability of  $\nu_e$  transitions into any other state [22] and for the size of CP or T violation that could be measured in long-baseline experiments in the  $\nu_\mu \rightleftharpoons \nu_e$  and  $\bar{\nu}_\mu \rightleftharpoons \bar{\nu}_e$  channels [23].

Let us consider now the effective Majorana mass in neutrinoless double- $\beta$  decay,

$$|\langle m \rangle| = \left| \sum_{k=1}^4 U_{ek}^2 m_k \right|. \quad (6.1)$$

In scheme A, since  $c_e$  is small, the effective Majorana mass is approximately given by

$$|\langle m \rangle| \simeq |U_{e3}^2 + U_{e4}^2| m_4 \simeq |U_{e3}^2 + U_{e4}^2| \sqrt{\Delta m_{\text{LSND}}^2}. \quad (6.2)$$

Therefore, in scheme A the effective Majorana mass can be as large as  $\sqrt{\Delta m_{\text{LSND}}^2}$  [29–32]. On the other hand, in scheme B the contribution of the “heavy” neutrino masses  $m_3$  and  $m_4$  to the effective Majorana mass is strongly suppressed [29–32]:

$$|\langle m \rangle|_{34} \equiv |U_{e3}^2 m_3 + U_{e4}^2 m_4| \lesssim c_e m_4 \leq a_e^0 \sqrt{\Delta m_{\text{LSND}}^2}. \quad (6.3)$$

Finally, if the upper bound  $N_\nu^{\text{BBN}} < 4$  for the effective number of neutrinos in Big-Bang Nucleosynthesis is correct [35], the mixing of  $\nu_s$  with the two mass eigenstates responsible for the oscillations of atmospheric neutrinos must be very small [24–26]. In this case atmospheric neutrinos oscillate only in the  $\nu_\mu \rightarrow \nu_\tau$  channel and solar neutrinos oscillate only in the  $\nu_e \rightarrow \nu_s$  channel. This is very important because it implies that the two-generation analyses of solar and atmospheric neutrino data give correct information on neutrino mixing in the two four-neutrino schemes A and B. Otherwise, it will be necessary to reanalyze the solar and atmospheric neutrino data using a general formalism that takes into account the possibility of simultaneous transitions into active and sterile neutrinos in solar and atmospheric neutrino experiments [33].

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